Robust Tracking Controller Design With Uncertain Friction Compensation Based on a Local Modeling Approach

Lotfi Mostefai, Mouloud Denaï, and Yoichi Hori, *Fellow, IEEE*

***Abstract*—This paper presents a new methodology for the design of a robust controller to compensate for friction-induced dynamical characteristics inherently present in servodrives systems. A friction model is developed using a local modeling approach of the physical properties of friction along the operating range of the underlying system. Generally, developing a faithful model for physical nonlinearities is still a challenging task that is strongly related to the identification effort required by the structure of the model and the complexity of the control algorithm. The proposed model has the advantage of being simple and able to describe friction locally. The accuracy of the estimator based on the model structure can be improved by a gain-scheduled input signal obtained for different velocities and used as a precompensator of nonlinear friction. This leads to an effective linearzing strategy of the controlled system that subsequently simplifies the controller implementation stage. A stabilizing-state feedback controller is designed, assuming an inexact compensation of friction, which guarantees robustness against uncertainties arising from modeling errors and achieves high tracking performance of the overall controlled system. Experimental tests performed on a robot joint laboratory prototype demonstrate the effectiveness of the proposed friction compensation scheme to improve the performance of the overall system.**

***Index Terms*—Friction compensation, gain-scheduling control, linear matrix inequality (LMI), local modeling, state feedback.**

I. Introduction

The field of motion control engineering is still facing many potentially challenging problems arising from diverse nonlinear characteristics, such as friction, backlash, and hysterisis, which are the principal cause of performance deterioration, wear, and even instabilities of real-world mechanical systems [1]. Friction is an inherently complex multifaceted phenomenon that depends on many factors such as displacement and relative velocities of the bodies, properties of the surface materials in contact, temperature, etc. Friction can induce undesirable effects like tracking errors, stick–slip motions, and limit cycles that can be particularly critical for certain systems.

Several friction estimation and compensation strategies have been proposed in the literature. The considered methods incorporate different friction models using observer-based control [2], adaptive friction compensation [3], [4], sliding mode control [5], and neural and fuzzy control [6], [7]. Some authors have also addressed the problem of inexact friction compensation and its effects on the controlled servosystem [8]. Basically, these underlying approaches can be regarded as either model-based techniques requiring a modeling-identification effort [9], or nonmodel-based methods where friction is considered as an unknown disturbance [10], even if a prior knowledge of friction is required since this latter is a part of the system dynamics [11].

Dynamic models have been essentially developed to give a better description of friction phenomena in mechanical systems characterized by the following physical properties.

1) *Presliding displacement:* This is a motion that appears during stiction with contact deformation at zero velocity where friction is only a function of the displacement
2) *Frictional memory:* This effect takes the form of hystersis loops relating friction to input velocities.

In the field of Tribology, as well as in the control community, several dynamic models have been developed to describe friction behavior [12]. The pioneering work of Dahl [13] in the field was the starting point for the majority of models proposed later. These analytic models are more or less complex and descriptive of friction phenomena. Recently, with the growing interests in motion control, friction models such as LuGre [14], Leuven [15], and many others [16], [17] have emerged. All these dynamic models claim fidelity in replicating friction behavior. However, there is little consensus in the literature on which specific model is best descriptive of friction phenomena. The precision required in the context of friction compensation is associated with a considerably extensive identification effort due to model complexity. Furthermore, the design and implementation of control algorithms based on such models become inevitably more and more complicated.

A general form of a dynamic friction force depending on position x, velocity v, and having only one hidden state z can be expressed as follows:

$$F = f(z, x, v). \tag{1}$$

The differential equation driving the internal state dynamics is given by

$$\dot{z} = g(z, x, v) \tag{2}$$

L. Mostefai was with the Institute of Industrial Science, The University of Tokyo, Tokyo 153-8505, Japan. He is now with Mohamed Boudiaf University of Science and the Technology of Oran, Oran 31000, Algeria (e-mail: lotfi@horilab.iis.u-tokyo.ac.jp).

M. Denaï is with the Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield, S1 3JD, U.K., on leave from Mohamed Boudiaf University of Science and the Technology of Oran, Oran 31000, Algeria (e-mail: m.denai@sheffield.ac.uk).

Y. Hori is with the Institute of Industrial Science, The University of Tokyo, Tokyo 153-8505, Japan (e-mail: hori@iis.u-tokyo.ac.jp).

where f and g are nonlinear functions that may also include hybrid dynamics to describe the natural complexity of friction [18].

The estimation of the friction model parameters is another challenging part strongly related to the model structure, as discussed in [1].

This paper presents a new model structure for the estimation and robust compensation of friction dynamics. An identification procedure is developed and a highly accurate tracking controller is then designed under uncertain compensation assumptions based on linear matrix inequality (LMI) optimization techniques. The local modeling approach adopted in this paper is not claimed to faithfully reproduce the overall friction complex dynamics; however, it provides a simple and efficient methodology for the design of a robust controller that achieves high tracking performance. The proposed technique has been extensively evaluated in simulations and validated experimentally on a robot joint.

II. Formulation of Local Modeling Approach

A. Local Modeling Approach of Friction

A mechanical system with friction is governed by the following equation of motion:

$$m\frac{dv}{dt} = u - F \tag{3}$$

where m is the inertia, v the velocity, u the control signal, and F represents the friction forces of the system. This model will be subsequently used to describe the robot joint dynamics by neglecting Coriolis, centrifugal, and gravity forces.

The friction forces given by the nonlinear complex function F in (1) can be described by local linear dynamics according to the velocity-related operating conditions of the system governed by (3)

$$F = c_i z + d_i v \quad \text{for} \quad v \in \Omega_i. \tag{4}$$

Basically, F can be seen as the sum of a stiff and damping force where c_i can be treated as a local coefficient of stiffness, since z has the dimension of a displacement and d_i as the local damping factor, respectively. The expression ($\Omega_i \subset \Omega$) is defined as a bounded convex set of operating velocities that must depend on the variation of friction dynamics in order to fit the behavior of the real system. It should be emphasized that, generally, the size of the set depends on how fast are the dynamics of the nonlinearity. It is well known from the steady-state characteristics that real friction is highly nonlinear at very low velocities, where the number of sets i should be higher. In the same way and using simple approximations, (2) can be rewritten as

$$\dot{z} = -a_i z + b_i v, \qquad \text{for } v \in \Omega_i \text{ with } i = 0, 1, \ldots, n \tag{5}$$

where a_i and b_i are positive quantities defined according to the local behavior of the system with friction. Along the operating range set Ω_i, (5) is characterized by a static amplification term b_i/a_i and the time constant $1/a_i$. At very low velocities defined in the domain Ω_0, where the system is under microsliding

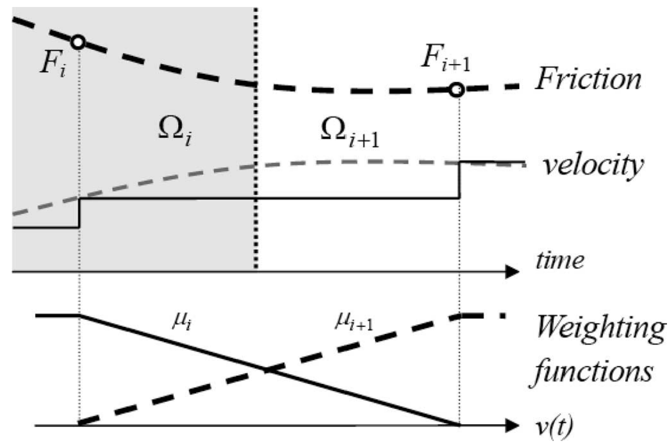


Fig. 1. Identification procedure chart based on local modeling approach.

motion, the dynamics of the friction force (6) can be reduced to

$$\begin{aligned} F &= c_0 z \\ \dot{z} &= v \end{aligned}, \qquad \text{for } v \in \Omega_0. \tag{6}$$

A Comparison of (4) and (5) with (6) yields the parameter $b_i = 1$, and $c_i = c_0$ can be defined inside Ω_0.

At higher velocities regime, and assuming that the system is operating at almost constant velocities resulting from slow-varying inputs, the steady-state condition can be satisfied ($\dot{z} \approx 0$), and the friction force dynamics are then obtained by solving the set of equations (4) and (5) for a constant input velocity inside the switching area of two successive domains Ω_i and Ω_{i+1}. Around a zero velocity, the level of friction is decided by its steady-state value as

$$F_i = \left(b_i\frac{c_i}{a_i} + d_i\right)v, \qquad \text{for } v \in \{\Omega_{1^-}, \Omega_{1^+}\} \tag{7}$$

where $\{\Omega_{1^-}, \Omega_{1^+}\}$ represent the first domain around zero velocity for negative and positive directions, respectively. From (7), an identification method is developed to define the parameter a_i for higher velocities using the dynamics of every two adjacent domains (see Fig. 1).

B. Friction Identification Procedure Based on Local Modeling

The proposed model given by (4) and (5) describes friction dynamics inside a certain set Ω_i. For simplicity, $b_i = 1$, $c_i = c_0$, and $d_i = d_0$ are kept constant without losing the capability of the model to describe the main friction features in the whole Ω domain. According to (7), a_i clearly has an influence on both the level and the speed convergence of the friction characteristic; a_i will be defined to fit the local level of friction for the assumed steady-state conditions inside each set Ω_i. Using a low-frequency sine-wave input signal, the system can be identified in slow motions so that at $t = 0$, the pair ($v = v_i$, $F = F_i$) is known from the experimental steady-state curve of friction and (7) is solved for $F = F_{i+1}$ to yield a_i. For the next adjacent set, $F_i(t) = F_{i+1}(t)$ is solved to determine which a_{i+1} can be calculated at $t = t_f$, time at which the final value of friction

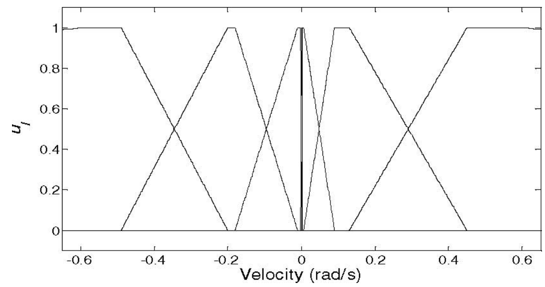


Fig.2. Weighting functions for the dynamic switching of local models.

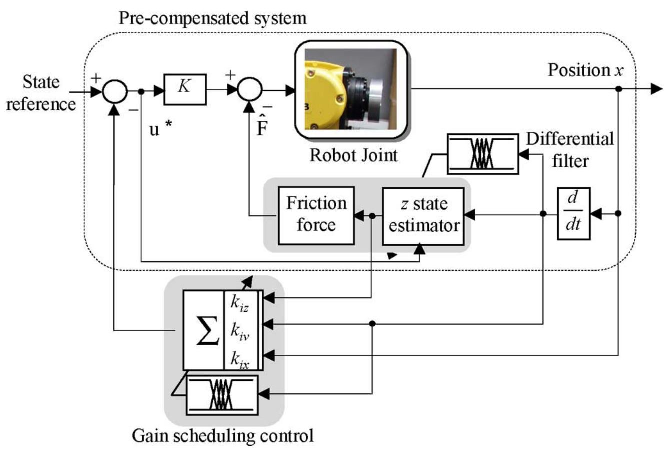


Fig.3. Robust gain-scheduling control design for friction compensation, overall scheme.

force in the set Ω_i equals to the initial value in the set Ω_{i+1}

$$\left(\frac{c_0}{a_{i+1}}\left(1-e^{-a_{i+1}(t_f)}\right)+d_0\right)v_{i+1}$$
$$=\left(\frac{c_0}{a_i}\left(1-e^{-a_i(t_f)}\right)+d_0\right)v_i. \quad (8)$$

Note that a_i is bounded according to the operating velocities of the system defined by $-\frac{F_{\min}}{c_0}\geq\frac{|v_i|}{a_i}\geq-\frac{F_{\max}}{c_0}$.

Obviously, the friction force exhibits faster variations at very low velocities, where a_i assumes smaller values, and therefore, more models are needed to reproduce the general behavior of the phenomena.

Smooth transition from a domain to another is ensured by including switching functions to effectively interpolate the model dynamics along the overall operating range Ω; this is achieved using weighting functions μ_i, as illustrated by Fig.2.

III. Robust Friction Compensator Design

The system with nonlinear friction given by (3)–(5) can now be described locally by a set of linear state-space equations inside each set Ω_i.

The advantage of the local modeling approach is that it allows the use of linear design methods inside the sets Ω_i. The overall control system is shown in Fig.3. The first component implements the cancellation of friction effects by adding a signal compensator in a minor local loop. The second component represents the design of a stable tracking controller that ensures

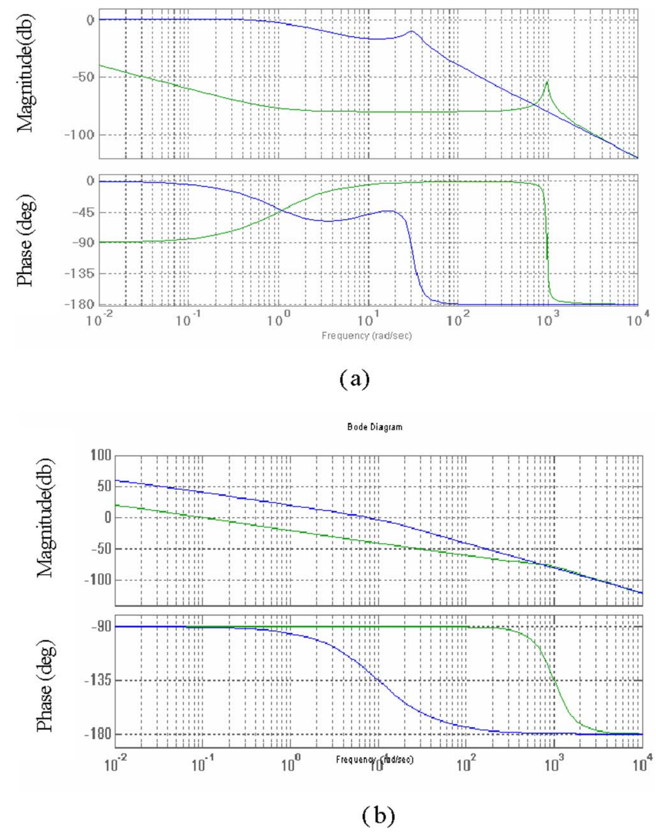


Fig.4. Bode diagram of local model of system before and after compensation. (a). At very low velocities (before: grey, after: dark). (b). Higher velocities (before: gray, after: dark).

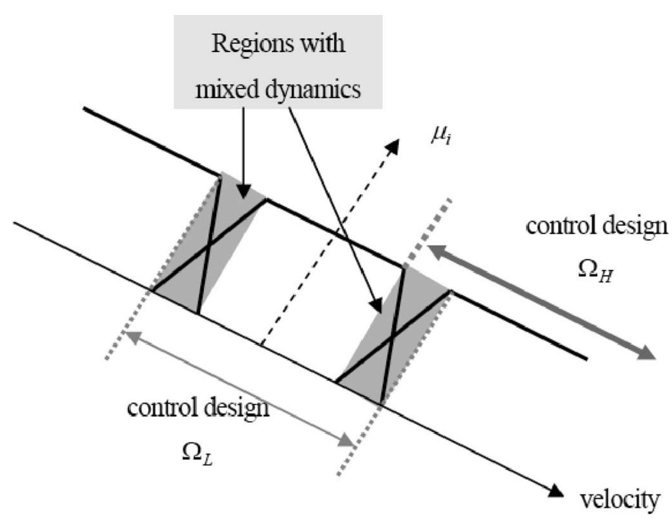


Fig.5. Gain-scheduling design based on local modeling approach.

robustness against disturbances and uncertain friction effects resulting from the first component.

A. Local-Models-Based Compensator Design

The general idea of the compensating technique is the rejection of the inverse of the nonlinearities to cancel their effects on the controlled system.

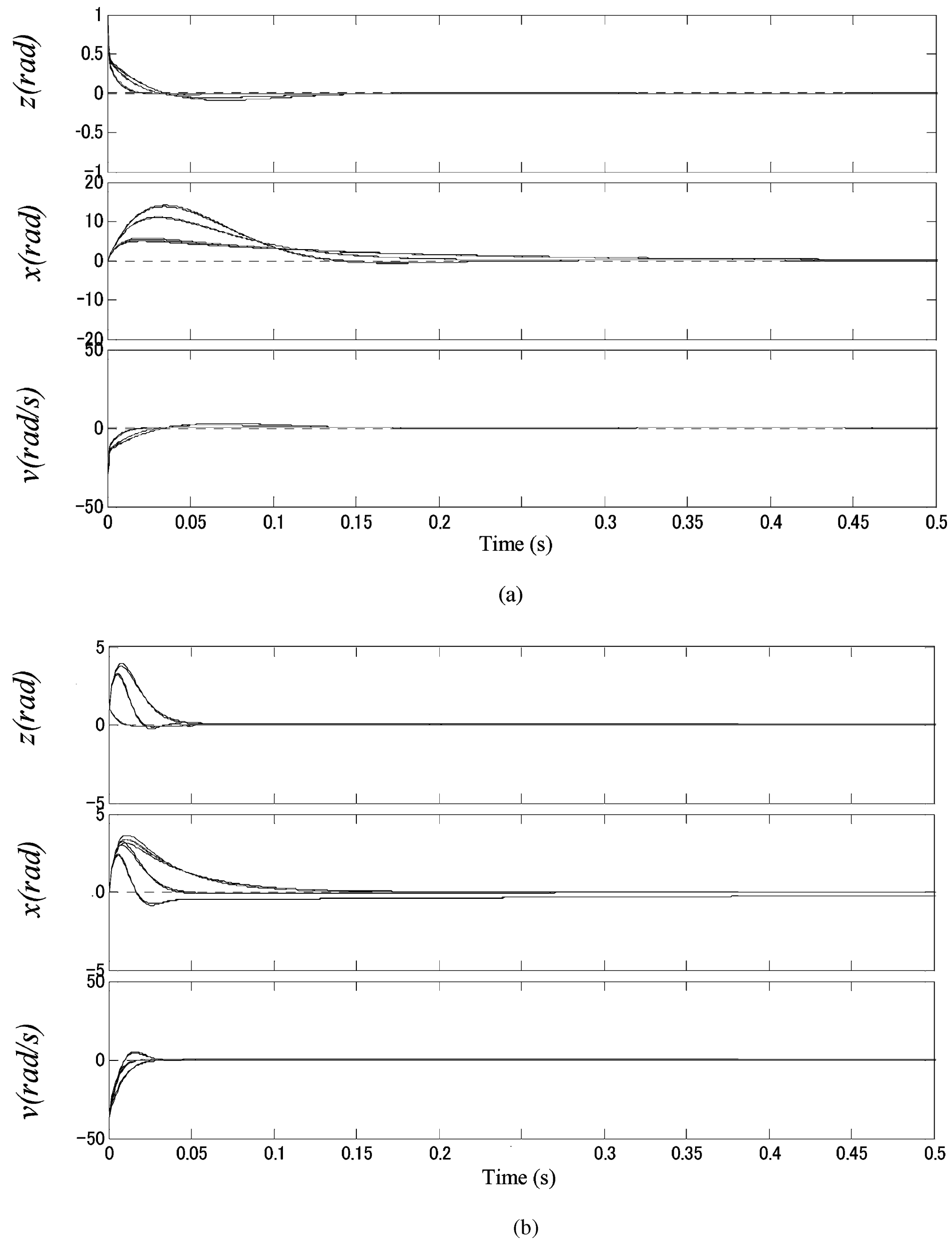


Fig. 6. Impulse responses using the bound values of the identified design parameters, under robust control. (a). At very low velocities Ω_L. (b). Higher velocities Ω_H.

Since in our case, the overall nonlinearity is described by a set of piecewise-linear elements inside a certain domain Ω_i, and the design of a stable state estimator of the nonlinearity inside each domain becomes straightforward. The proposed friction state estimator uses the dynamics (5) and takes the form

$$\dot{\hat{z}} = -a_i \hat{z} + v + l_i u^* \tag{9}$$

where l_i is a local compensating gain and $u^* = u/K$ is defined as the new control input of the compensated system and will include the tracking error information in the overall controlled system of Fig. 3. K is a fixed positive control gain, which will be set 0.1 later in the closed-loop controller design stage. Let the error in the friction state estimation be defined by

$$e_z = z - \hat{z}. \tag{10}$$

The dynamics of the considered system with friction (3)–(5) can then be changed by adding the new local compensating term (9), and using the error equation (10) that leads to the equation governing the precompensated system

$$\begin{aligned} \dot{e}_z &= -a_i e_z + l_i u^* \\ m\dot{v} &= K u^* - \Delta c_i e_z - \Delta d_i v \end{aligned}, \qquad \text{for } v \in \Omega_i \tag{11}$$

where $\Delta c_i = c_i - \hat{c}_0$ and $\Delta d_i = d_i - \hat{d}_0$ represent a measure of compensation mismatch in stiff and viscous friction force inside a local set Ω_i, respectively. At this stage, Δc_i is assumed to be slightly different for each one of the regions Ω_i, and has its maximum value around zero velocity range [-0.005, $+0.005$]. When designing the controller for the worst-case conditions, Δc_i will be fixed at its maximum value. Equation (11) allows the transformation of the initial system with friction such that a gain-scheduled control input is introduced into the friction estimation mechanism. A comparison of the local models frequency responses before and after compensation is shown in Fig. 4. The term added in (11) is used to shape the open-loop system with friction. The results demonstrate a considerable improvement in the local dynamics and a reduction in the effect of the nonlinearity that allows the design of a local robust control strategy. The local stability of the compensated system can be investigated with respect to the chosen gains l_i. One should note that at zero velocity, an additional position-dependent term is added to ensure a flat response in the low-frequency range.

B. LMI-Based Robust Gain-Scheduled Controller

The proposed modeling approach applied to local friction compensation allows our control problem to be reformulated and regarded as a control of an uncertain polytopic model [19]. Thus, the resulting mathematical model takes into account disturbances, uncertain compensation effects, and the variation of the design parameters in the compensator itself.

The feedback control u^* can then be synthesized using LMI techniques [19]. By eliminating z using (10), the dynamics of the precompensated system can be reformulated and written as follow:

$$\begin{aligned} \dot{\hat{z}} &= -\Delta a_i \hat{z} + v + w_z + l_i u^* \\ \dot{x} &= v \\ m\dot{v} &= K u^* - \Delta c_i \hat{z} - \Delta d_i v + w_v \end{aligned}, \qquad \text{for } v \in \Omega_i \qquad (12)$$

where w_z and w_v include external disturbances and the estimation error, which is locally bounded as a result of the design of the observer gains in (11).

Our objective is to design an optimal state feedback controller to achieve high tracking performance under the assumption of inexact friction compensation and in the presence of other disturbances. For such requirements to be fulfilled, and given the uncertain nature of the system described by local models, LMI control represents a suitable approach to deal with these multi-model suncertainties.

The control problem can be stated as follow: Design a stabilizing control law that guarantees performance for the system while taking into account all variations in the friction model parameters, observer gains, and uncertainties resulting from inexact compensation. This can be formulated through an LMI-based convex optimization procedure and stated as follows.

Find

$$u^* = k_z z + k_x x + k_v v \qquad (13)$$

1) that minimize $\|T\|_2$, which is the closed-loop H_2 norm of the transfer function T from w to $\xi = \alpha x + \beta u^*$, where α and β are weighting coefficient of position and input signal, respectively, and their choice is known to be related to performances criteria, as well as to the control signal that achieves such performances, this is termed the linear quadratic Gaussian (LQG) cost problem;

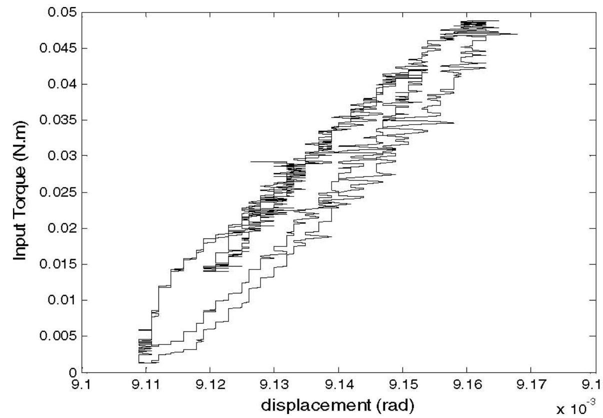


Fig. 7. Identified Dahl curve in robot joint, experimental results.

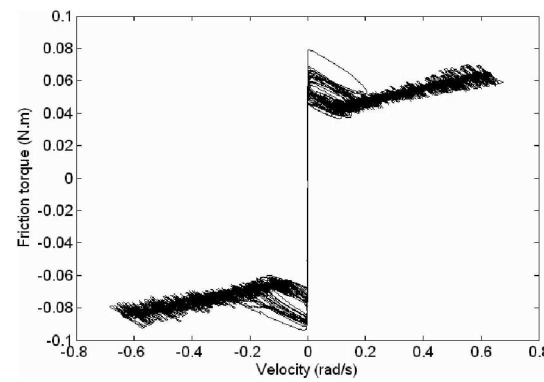


Fig. 8. Identified friction–velocity map in robot joint, experimental results.

2) all closed-loop poles lie inside the stable region with a maximum damping value of 0.1;

3) subject to the dynamics given by (12) inside all Ω.

Once the LMI problem is solved, the state feedback gains can be determined. Therefore, $\|T\|_2$ can be guaranteed not to exceed some predefined performance value ν, if there exist two symmetric matrices P and Q such that

$$\begin{aligned} &\begin{bmatrix} (A_i + B_{2i}K_{u*})P + P(A_i + B_{2i}K_{u*})^T & B_{i1} \\ B_{1i}^T & -I \end{bmatrix} < 0 \\ &\begin{bmatrix} Q & (C' + D'K_{u*})P \\ P(C' + D'K_{u*})^T & P \end{bmatrix} > 0 \\ &Trace(Q) < \nu^2 \end{aligned} \qquad (14)$$

where all LMI elements ensuring a robust local design are chosen to be as follows:

$$A_i = \begin{bmatrix} -\Delta a_i & 0 & 1 \\ 0 & 0 & 1 \\ \Delta c_i & \kappa & \Delta d_i \end{bmatrix}, \quad B_{1i} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad B_{2i} = \begin{bmatrix} l_i \\ 0 \\ K \end{bmatrix} \qquad (15)$$

$$C' = [0 \quad \alpha \quad 0], \qquad D' = [\beta]$$

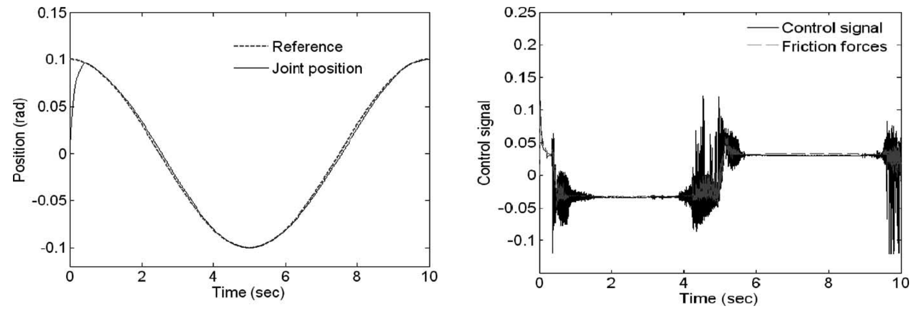


Fig. 9. Proposed control, joint position (left), and control signal and friction (right).

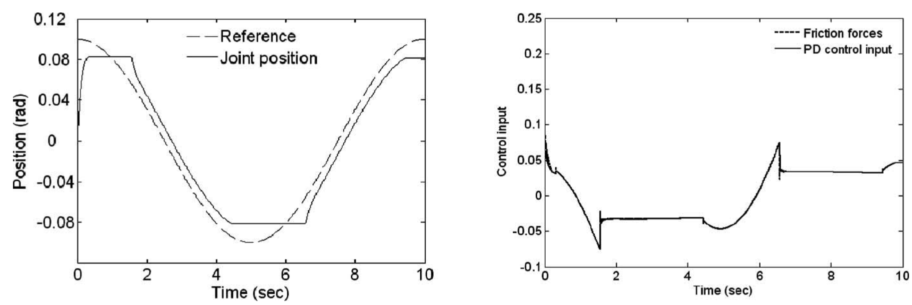


Fig. 10. PD control, joint position (left), and control signal and friction (right).

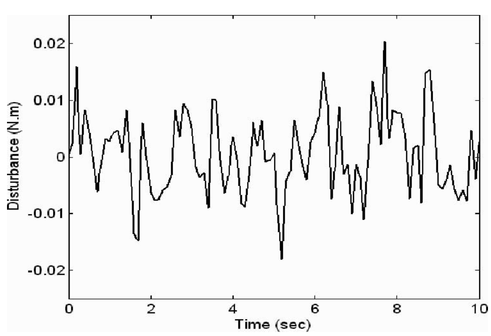


Fig. 11. Disturbance added to detune friction from its nominal level.

where κ is a small gain that provides a solution for all the operating range.

Regarding the nature of the tracking problem, the system will run into a severe regime, including reversal of velocities, where static friction has a major influence and stick–slip motions may occur for relatively higher velocities. It follows that the optimal controller for the reversed velocity condition is not necessarily the same for higher velocities where the dynamics of friction are slower. Hence, improved performances can be achieved by incorporating a gain-scheduling optimal controller. The optimal control problem (13) is then solved for two different overlapping regions Ω_L and Ω_H to generate two state feedback gains, where Ω_L is a symmetric set including zero velocity, and Ω_H is the higher velocity set satisfying the following two conditions: 1) $\Omega_L \cup \Omega_H = \Omega$ being the overall set and 2) $\Omega_L \cap \Omega_H = \Omega_m$ being the region of mixed dynamics corresponding to lower and higher velocities, as illustrated by Fig. 5.

The problem is converted into an LMI problem and solved using MATLAB's LMI control toolbox [20]. The impulse responses of the closed-loop control system from w to ξ are shown in Fig. 6 for both sets Ω_L and Ω_H.

Each curve is equivalent to a bound value for either a design parameter (l_i, Δa_i) or for one of the uncertainties in friction compensation (Δc_i, Δd_i).

From (12), (13), and the LMI solutions (14) for the adopted local partitioning described in Fig. 5, the overall gain-scheduling control law applied to the robot joint for friction compensation has the following expression:

$$u = Ku^* + \hat{F} = \mu_{\Omega L} K_{\mu_{\Omega L}} + \mu_{\Omega H} K_{\mu_{\Omega H}} + c_0 \hat{z} + d_0 v. \tag{16}$$

Robust control design ensures that, for any given uncertainty or design inside the bounds considered, all the responses will be enveloped inside the impulse response. Note that the same weighting function will be used in the gain-scheduling algorithm.

IV. Simulation Results

A. Identification for the Local Modeling Approach

The model developed earlier in its simplest structure is used for parameters identification of the robot joint [9]. The robot joint is initially excited with a sinusoidal input torque not exceeding the static friction level. While the system is running in its presliding regime, the Dahl curve can be plotted, as shown in Fig. 7, and the parameter c_0 can be estimated.

For higher velocities, the limited workspace of the robot joint prohibits the use of classical methods for friction compensation similar to those suggested in the literature [12]. Therefore, a

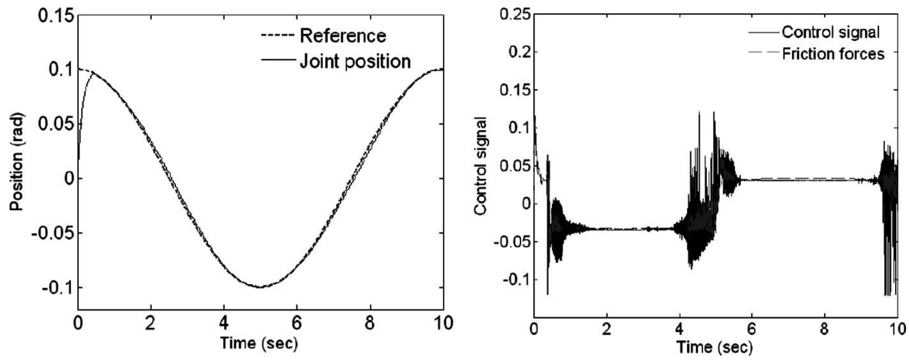


Fig. 12. Joint position, proposed control with disturbance.

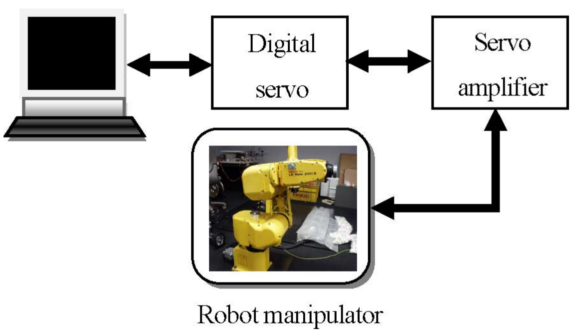


Fig. 13. Experimental setup.

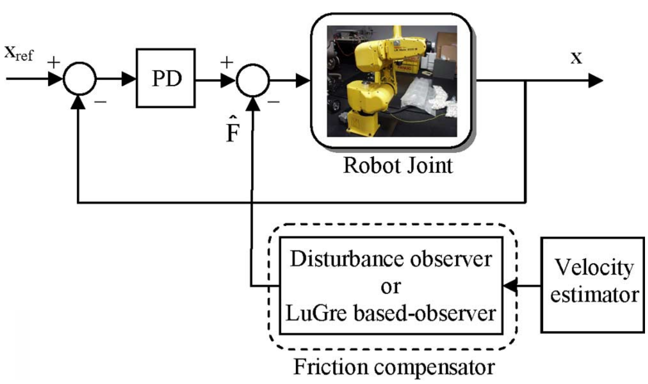


Fig. 14. Controlled robot joint methods for experimental results comparison.

more suitable method similar to that used in [9] was adopted. The method consists of forcing a proportional–derivative (PD) controlled joint motor to track a low-frequency sinusoidal position reference, and estimating the equivalent torques and velocities from position measurements. The identification data can be extracted from the equation of motion that governs the PD-controlled robot joint

$$m\dot{v} = u_{\mathrm{PD}} - F \tag{17}$$

where u_{PD} is the input control signal of the low-gain PD controller that is required to overcome friction forces in the robot joint. After a few trials, the amplitude and frequency of the input signal are selected, and subsequently, a friction–velocity map can be plotted, as shown in Fig. 8. For slow motions, $\dot{v} \approx 0$ and $F \approx u_{\mathrm{PD}}$.

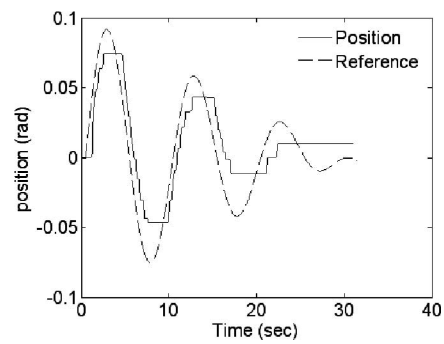


Fig. 15. Tracking under PD control. Experimental results.

B. Tracking Control Based on Local Friction Compensation

The performance of the proposed method is now evaluated in simulations and experimental environments. The LuGre model ($l_f = 0$) is used to generate real friction forces in the simulations, whereas the model-based observer is used in our experiments for comparison [14]

$$\begin{aligned} \dot{z} &= v - \frac{c_0}{g(v)}\,|v|\,z - l_f.u^* \\ F &= c_0 z + \sigma_1 \dot{z} + F_v v \end{aligned} \tag{18}$$

where $g(v) = F_C + (F_S - F_C)e^{v^2/v_s^2}$ is a Gaussian function with all parameters defined in Table I. This function is used to fit the steady-state friction curve. It should be noted that it is very difficult to reproduce accurately the Stribeck friction curve with $g(v)$. Moreover, the fact that friction can show considerable asymmetry between negative and positive velocity makes the proposed model and the associated estimator (12) an effective design strategy. A sinusoidal reference with $f = 0.1$ Hz is chosen to ensure the tracking of slow motions and reversal velocities, regions where friction has a considerable influence. Fig. 9 demonstrates a net improvement of the proposed method over a conventional PD control (see Fig. 10).

The robustness test is performed in simulations by adding a filtered white noise signal to friction that is assumed to detune the friction force level from its nominal value, as illustrated by Fig. 11. The results after compensation shown in Fig. 12 demonstrate the robustness of the proposed method.

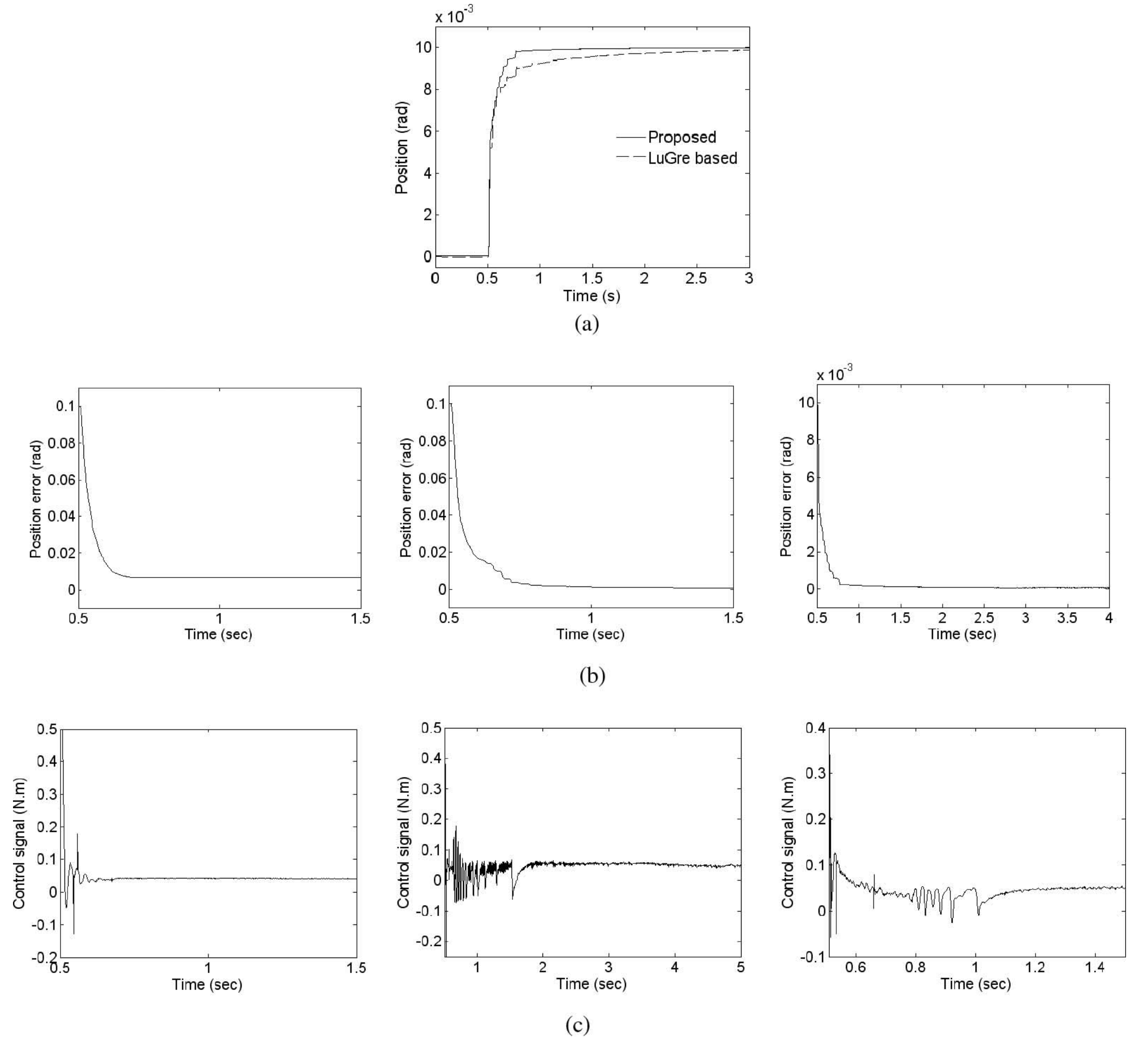


Fig. 16. Robot joint positioning 100 mrad. Experimental results. (a) Joint position for step reference 100 mrad. (b). Position error: PD control (left), LuGre-based method (middle), versus local modeling approach (right). (c). Control signal: PD control (left), LuGre-based method (middle), versus local modeling approach (right).

V. Real-Time Evaluation

The experimental setup shown in Fig. 13 uses a 700-MHz PC operating under RT-LINUX and a digital servo adapter that communicates via an optical cable to ensure noise-free data transfer. The control algorithms are implemented in C code with a sample time of $T_s = 0.001$ s.

Fig. 14 introduces a block diagram of the control methods used in the experiments for performances evaluation. Fig. 15 shows the response of the robot joint under PD control. A reference trajectory is generated as follows:

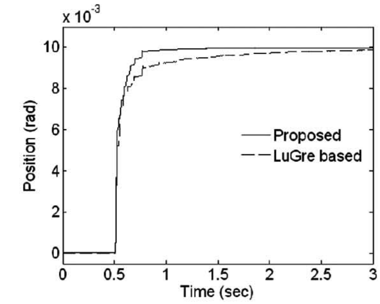
$$x_{\mathrm{ref}} = (0.1 - 0.0033t)\sin(0.2\pi t)\ \mathrm{rad}. \tag{19}$$

For this trajectory, the robot joint operates in different regimes at low speed and goes several times reversal through velocities. Large tracking errors have been obtained due to the dominance of friction at low regimes against the other nonlinear characteristics.

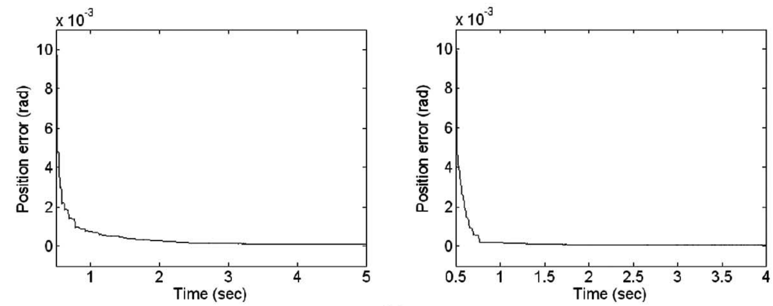
Initially, a step reference of 100 mrad is applied. As expected, a steady-state error exits under PD control, and the use of higher stiff gains and integral gain is required to improve the positioning performance. However, the use of a fixed gain may lead to control input saturation and even the generation of limit cycles. To overcome this, a nonlinear gain-scheduling strategy can be introduced [21].

Fig. 16 clearly indicates the improvement achieved after friction compensation. However, the performance of gain scheduling based on local modeling approach in term of time response and control input profile is slightly superior to that based on LuGre model, since it relies on optimal design.

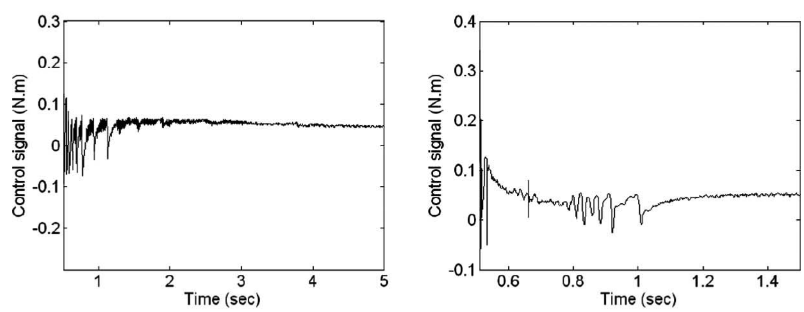
For a 10-mrad step reference, the LuGre-based compensator used in the previous experiment ($l_f = 0.35$) shows an oscillatory response that needs to be redesigned. A value of $l_f = 0.25$ produced better performance. The proposed method, on the other hand, achieved a clear improvement and better performances, as shown in Fig. 17, coping with the asymmetry of the friction force exhibited by the robot joint. This is due to



(a)



(b)



(c)

Fig. 17. Robot joint positioning 10 mrad. Experimental results. (a) Joint position for step reference 10 mrad. (b) Position error: LuGre-based method (left), versus local modeling approach (right). (c) Control signal: LuGre-based (left), versus local modeling approach (right).

the fact that instead of using a fixed controller structure, the local approach allows us to freely choose the observer gains inside each set separately to enhance the positioning performances without compromising the stability of the controlled system.

Further experiments have been conducted to verify the effectiveness of the proposed algorithm using the same reference (19). Two different approaches are applied to improve the tracking performance of the robot and compared with the proposed control method. The first one is a free model approach based on the disturbance observer developed in [22]. The second is based on a dynamic model and requires a precise model of friction for compensation. Therefore, for slow motions, where the level of friction is very hard to define accurately in Fig. 8, the proposed approach seems to be more effective in reducing friction induced errors, and this is performed in two stages: first, the major part is compensated by the minor loop, and the tracking error coming from the uncertain compensation is minimized by the gain-scheduled controller (16) depicted in Fig. 3. The calculated rms error clearly indicates the superiority of the proposed method with $e_{\mathrm{rms}} = 0.3781$ mrad over the disturbance observer-based control (PD $+$ DOB case) with $e_{\mathrm{rms}} = 0.6916$ mrad and friction-model-based compensator (PD $+$ LuGre case) where $e_{\mathrm{rms}} = 0.5006$ mrad. In case of tracking at slow motions in Fig. 18, the problem becomes more crucial and friction is more difficult to determine since the robot joint will be operating at very low-velocity range. Naturally, the disturbance observer shows some limitations around zero velocity, and LuGre-based compensator with its fixed structure was not able to completely compensate friction-induced errors. On the other hand, the proposed controller provides some flexibility to fix controller gains independently of the different velocity ranges.

It should be noted that chattering in the control signal is mainly generated by the friction compensation signal. This part of the controller structure depends essentially on the velocity, which is estimated and can have a direct influence on the quality of the control signal. Therefore, one possible way to make the

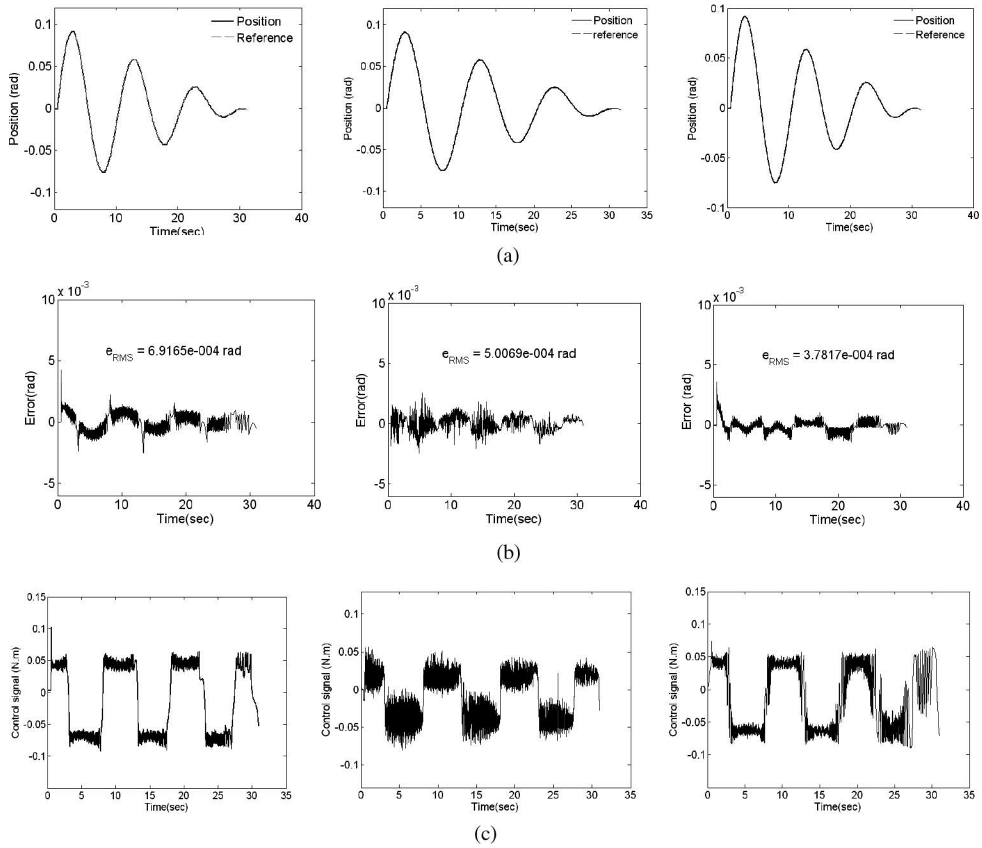


Fig. 18. Robot joint tracking $x_{\mathrm{ref}} = (0.1 - 0.0033t)\sin(2\pi ft)$ (in radians). Experimental results. (a). Joint position ($f = 0.1$ Hz), Experimental results comparison: PD + DOB control (left), LuGre-based (middle), proposed (right). (b). Tracking error ($f = 0.1$ Hz), Experimental results comparison: PD + DOB control (left), LuGre-based (middle), and proposed (right). (c) Control signal ($f = 0.1$ Hz), Experimental results comparison: PD + DOB control (left), LuGre-based (middle), and proposed (right).

control signal less noisy and reduce the chattering phenomenon would be to improve the estimation of the velocity. This can be achieved by a differentiation-low-pass filtering of the signal acquired from the position encoder. The time constant of the filter should be defined carefully to avoid compromising the quality of the estimation by introducing a reasonable delay time.

VI. Conclusion

The proposed control scheme relies on local identified parameters to design a friction compensator in a minor loop. A gain-scheduled robust controller is then synthesized under some severe assumptions such as uncertain compensation. Linear matrix inequalities were used to deal with the multimodel nature of the proposed method. Simulations and experimental results demonstrate the effectiveness of the proposed approach to solve the friction compensation problem. The proposed approach is based on a simplistic design methodology, and is proved to cope very well with the asymmetric nature of friction compared to other model-based friction compensation methods. Tracking trajectories for high velocities requires more local models that can increase the complexity of the design regarding the number of LMIs to be solved. This method is suitable for many applications including vehicle stability control, hard disc drive (HDD), and piezoelectrically actuated HDD.

Appendix

TABLE I
Simulation and Design Parameters

F_S (N.m)	0.07
F_c (N.m)	0.045
F_v	0.056
v_s (m/s)	0.1
σ_0	950
σ_1	1.0
m	0.001
K_P	2.50
K_D	0.12
K_x, K_v, K_z for Ω_L	0.017,0.241,1574.9
K_x, K_v, K_z for Ω_H	21.615,2.104,197
K	0.1
v_i (rad/s)	-0.5,-0.2,-0.01,-0.001,0,0.001,0.05,0.12,0.5
K_i	0,-0.5,-1.5,-2.5,-1.5,0.5,0

REFERENCES

[1] B. Armstrong-Helouvry, *Control of Machines With Friction*. New York: Kluwer, 1991.

[2] P. Vedagarbha, D. M. Dawson, and M. Feemster, "Tracking control of mechanical systems in the presence of nonlinear dynamic friction effects," in *Proc. Amer. Control Conf.*, 1997, pp. 2284–2289.

[3] S. S. Ge, T. H. Lee, and S. X. Ren, "Adaptive friction compensation of servomechanisms," *Int. J. Syst. Sci.*, vol. 32, no. 4, pp. 523–532, 2001.

[4] C. Du, L. Xie, J. Zhang, and G. Guo, "Disturbance rejection for a data storage system via sensitivity loop shaping and adaptive nonlinear compensation," *IEEE/ASME Trans. Mechatronics*, vol. 13, no. 5, pp. 493–501, Oct. 2008.

[5] G. Song, L. Gai, Y. Wang, and R. W. Longman, "A sliding-mode based smooth adaptive robust controller for friction compensation," *Int. J. Robust Nonlinear Control*, vol. 8, pp. 725–739, 1998.

[6] R. R. Selmic and F. L. Lewis, "Neural-network approximation of piecewise continuous functions: Application to friction compensation," *IEEE Trans. Neural Netw.*, vol. 13, no. 3, pp. 745–751, May 2002.

[7] Y. Tang and D. Velez-Diaz, "Robust fuzzy control of mechanical systems," *IEEE Trans. Fuzzy Syst.*, vol. 11, no. 3, pp. 411–418, Jun. 2003.

[8] C. C. de Wit, "Robust control for servo-mechanisms under inexact friction compensation," *Automatica*, vol. 29, no. 3, pp. 757–761, 1993.

[9] M. R. Kermani, R. V. Patel, and M. Moallem, "Friction Identification and compensation in robotic manipulators," *IEEE Trans. Instrum. Meas.*, vol. 56, no. 6, pp. 2346–2353, Dec. 2007.

[10] Y. Hong and B. Yao, "A globally stable high-performance adaptive robust control algorithm with input saturation for precision motion control of linear motor drive systems," *IEEE/ASME Trans. Mechatronics*, vol. 12, no. 2, pp. 198–207, Apr. 2007.

[11] M. Iwasaki, T. Shibata, and N. Matsui, "Disturbance observer-based nonlinear friction compensation in table drive system," in *Proc. 5th Int. Workshop Adv. Motion Control*, 1998, pp. 299–304.

[12] B. Armstrong-Helouvry, P. Dupont, and C. C. de Wit, "A survey of models, analysis tools and compensation methods for the control of machines with friction," *Automatica*, vol. 30, no. 7, pp. 1083–1138, 1994.

[13] P. Dahl, "A solid friction model," Aerospace Corporation, El Segundo, CA, Tech. Rep. TOR-0158(3107–18), 1968.

[14] C. C. de Wit, H. Olsson, K. J. Astrom, and P. Lischinsky, "A new model for control of systems with friction," *IEEE Trans. Autom. Control*, vol. 40, no. 3, pp. 419–425, Mar. 1995.

[15] J. Swevers, F. Al-Bender, C. G. Ganseman, and T. Prajogo, "An integrated friction model structure with improved presliding behavior for accurate friction compensation," *IEEE Trans. Autom. Control*, vol. 45, no. 4, pp. 675–686, Apr. 2000.

[16] G. Ferreti, G. Magnani, and P. Rocco, "Single and multistate integral friction models," *IEEE Trans. Autom. Control*, vol. 49, no. 12, pp. 2292–2297, Dec. 2004.

[17] F. Al-Bender, V. Lampaert, and J. Swevers, "The generalized Maxwell-slip model: A novel model for friction simulation and compensation," *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1883–1887, Apr. 2005.

[18] D. A. Haessig and B. Friedland, "On the modelling and simulation of friction," *Trans. ASME, J. Dyn. Syst. Meas. Control*, vol. 113H3I, pp. 354–362, Sep. 1991.

[19] P. P. Khargonekar and M. A. Rotea, "Mixed H_2/H_∞ control: A convex optimization approach," *IEEE Trans. Autom. Control*, vol. 39, no. 7, pp. 824–837, Jul. 1991.

[20] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolobox for Use With MATLAB*. Natick, MA: The Mathworks, Inc., 1995.

[21] B. Armstrong, D. Neevel, and T. Kusik, "New results in NPD control: Tracking, integral control, friction compensation and experimental results," presented at the Int. Conf. Robot. Autom., Detroit, MI, 1999.

[22] T. Umeno and Y. Hori, "Robust speed control of DC servomotors using modern two degrees-of-freedom controller design," *IEEE Trans. Ind. Electron.*, vol. 38, no. 5, pp. 363–368, Oct. 1991.

Lotfi Mostefai received the Master's degree in electrical control engineering from Mohamed Boudiaf University of Science and Technology, Oran, Algeria, in 2002.

He was a Lecturer at the University Moulay Tahar, Saida, Algeria, where he was engaged in delivering lectures on "control engineering" and "microprocessors and microcontrollers in industrial applications." In 2006, he joined HORILAB in the Institute of Industrial Science, The University of Tokyo, Tokyo, Japan, as a Research Student. His current research interests include optimal and robust control, motion control systems, smart materials and their applications, automotive engineering, and renewable energies.

Mouloud Denaï received the Bachelor's degree in electrical engineering from the University of Algiers, Algiers, Algeria, in 1982, and the Ph.D. degree in control engineering from the University of Sheffield, Sheffield, U.K., in 1988.

He is currently a Professor at Mohamed Boudiaf University of Science and the Technology of Oran, Oran, Algeria. He is currently on research leave in the Department of Automatic Control and Systems Engineering, University of Sheffield, Sheffield, U.K. His research interests include intelligent control design, intelligent fault detection in power systems, advanced control of power devices, modeling and control of electric power systems, and modeling, simulation, and control design for efficiency optimization in the field of renewable energies such as solar, wind, investigation of power electronics interface for renewable energy systems, modeling and control of life sciences systems, data mining, and knowledge discovery for system modeling.

Yoichi Hori (S'81–M'83–SM'00–F'05) was born on July 14, 1955. He received the B.S., M.S., and Ph.D. degrees in electrical engineering from The University of Tokyo, Tokyo, Japan, in 1978, 1980, and 1983, respectively.

In 1983, he was a Research Associate in the Department of Electrical Engineering, University of Tokyo, where he successively became an Assistant Professor, an Associate Professor, and a Professor in 2000. In 2002, he joined, as a Professor, the Department of Electrical Control System Engineering, Information and Electronics Division, Institute of Industrial Science, The University of Tokyo. From 1991 to 1992, he was a Visiting Researcher at the University of California, Berkeley. His current research interests include control theory and its industrial application to motion control, mechatronics, robotics, electric vehicle, etc.

Prof. Hori was the Treasurer of the IEEE Japan Council and the Tokyo Section during 2001–2002. He is now an AdCom member of the IEEE Industrial Electronics Society (IES). He was the Vice President of the Institute of Electrical Engineers of Japan (IEEJ) Industry Applications Society in 2004–2005. He has been the Chairman of the Energy Capacitor Storage System Forum, and the Chairman of the Motor Technology Symposium of Japan Management Association. He is a member of the IEEJ Japan Society of Mechanical Engineers, Systems Information Computer Ergonomics, Robotics Society of Japan, Society of Automotive Engineers of Japan, Japan Society for Simulation Technology, etc. He received Best Transactions. Paper Awards from the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS in 1993 and 2001.